

# MEDIUM- $\beta$ SUPERCONDUCTING ACCELERATING STRUCTURES

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**Jefferson Lab**

**Spoke Resonator Workshop**  
**Los Alamos**

**7-8 October 2002**

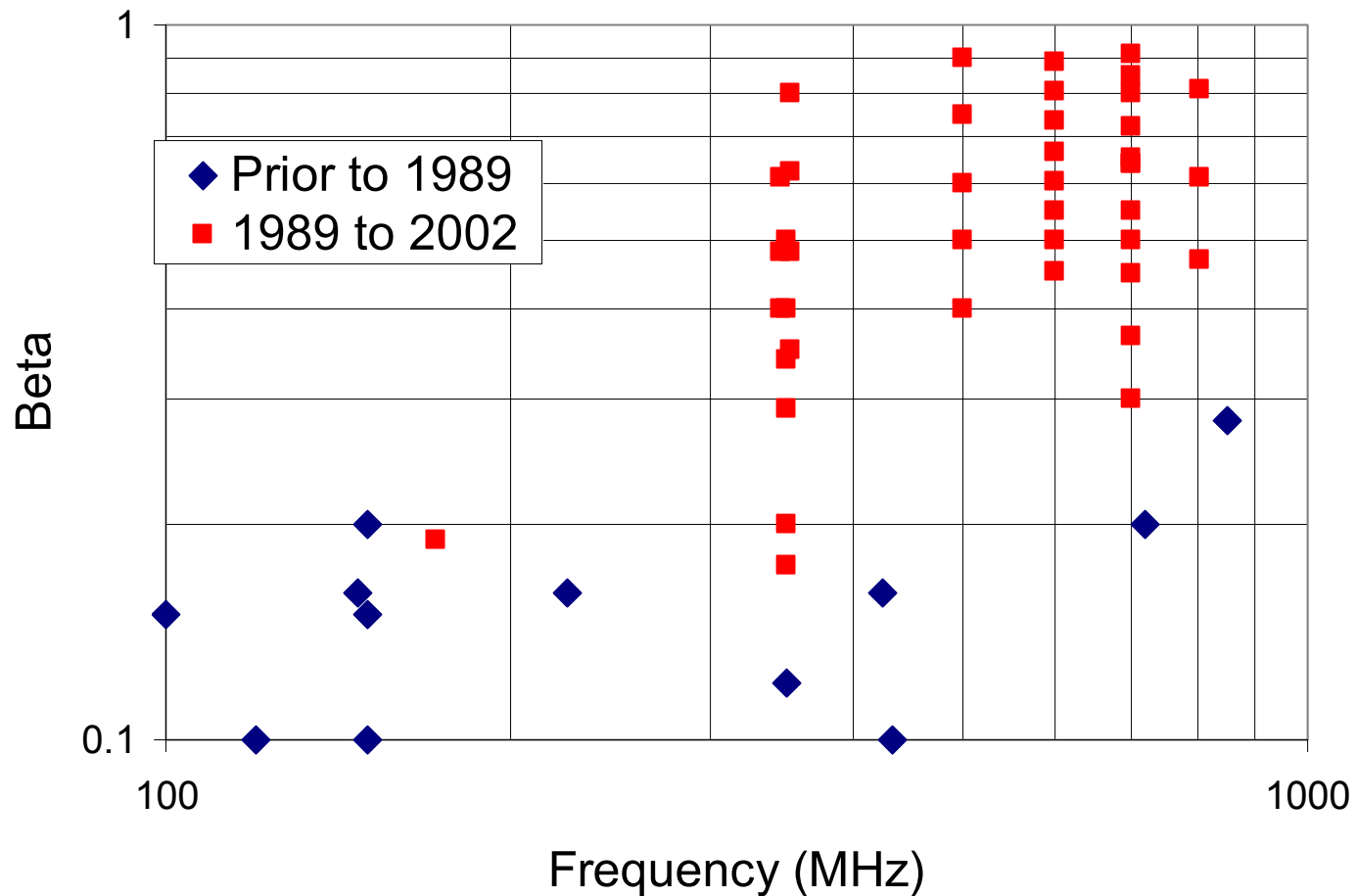


# Outline

- . Historical background
- . Basic geometries
- . Survey of properties
- . Summary



# $\beta < 1$ Superconducting Structures – 2002



# Basic Structure Geometries

- **Resonant Transmission Lines**

- $\lambda/4$

- Quarter-wave
    - Split-ring
    - Twin quarter-wave
    - Lollipop

- $\lambda/2$

- Coaxial half-wave
    - Spoke
    - H-types

- **TM**

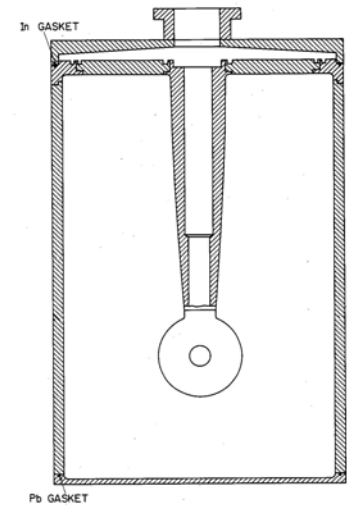
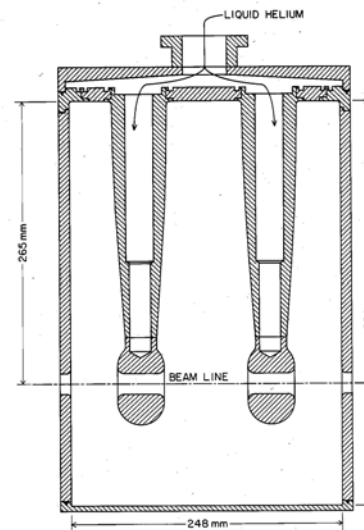
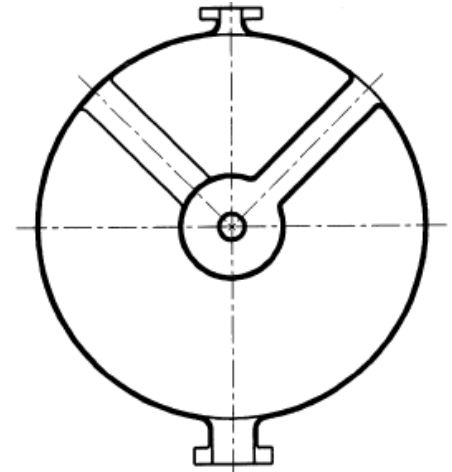
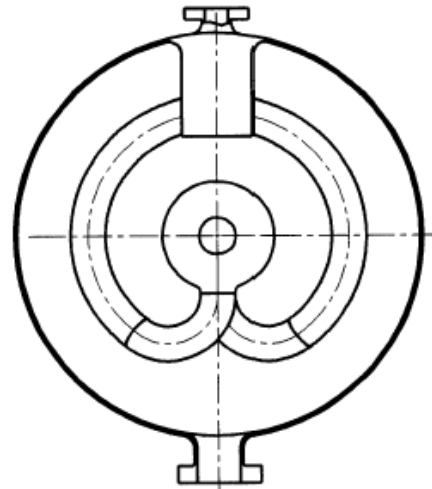
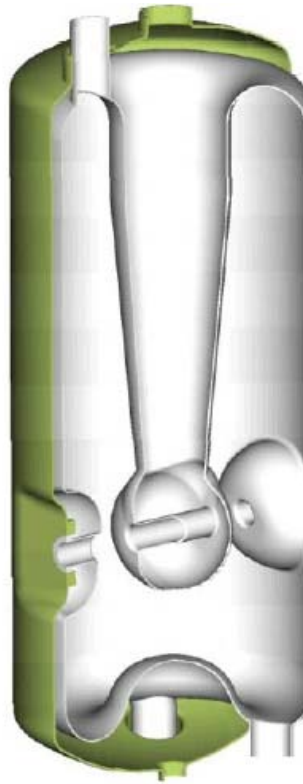
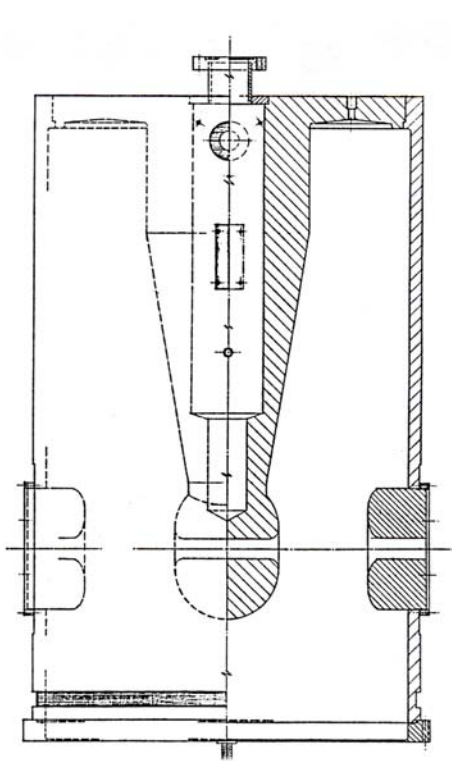
- Elliptical
  - Reentrant

- **Other**

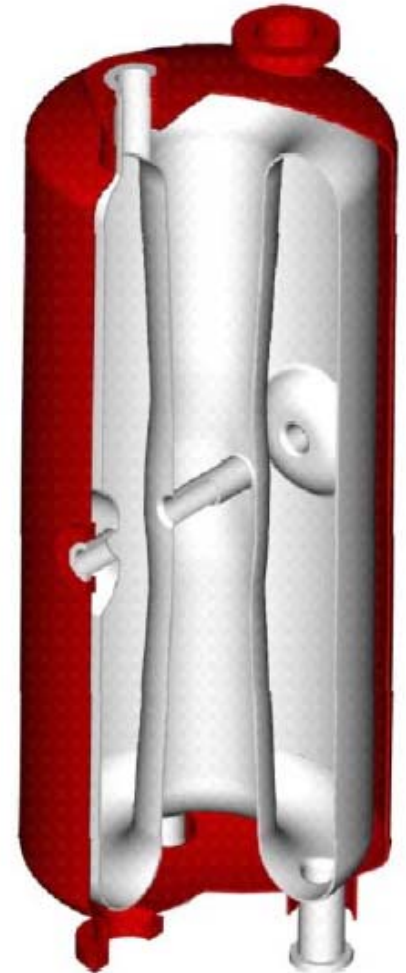
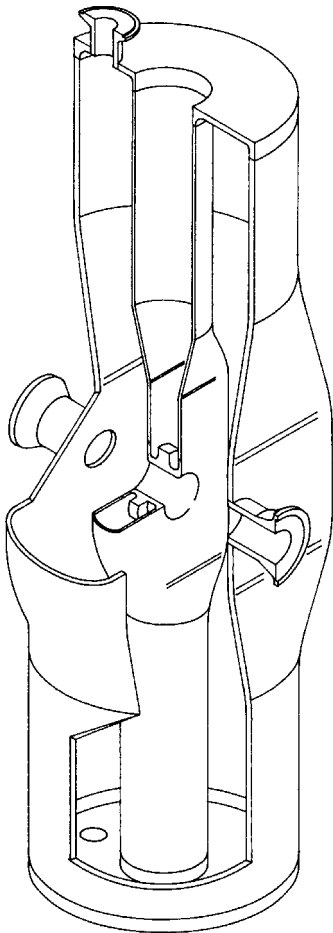
- Alvarez
  - Slotted-iris



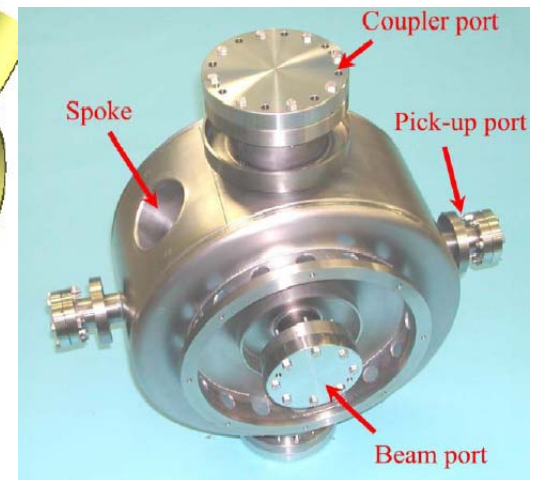
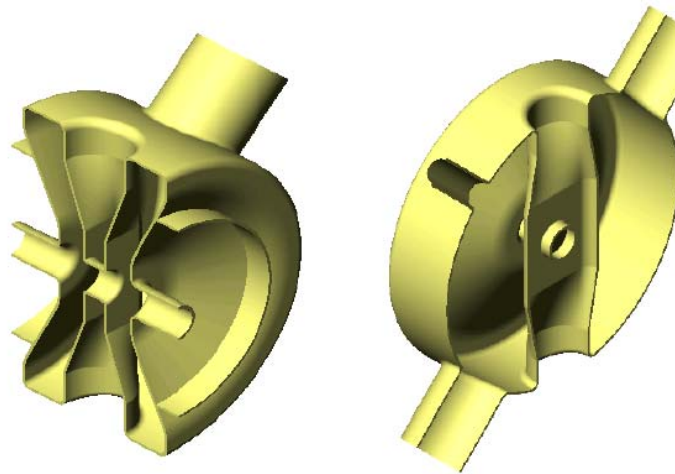
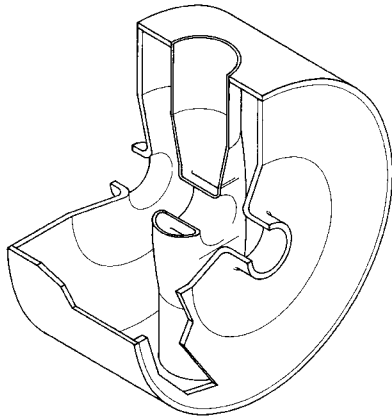
# $\lambda/4$ Resonant Lines



# $\lambda/2$ Resonant Lines

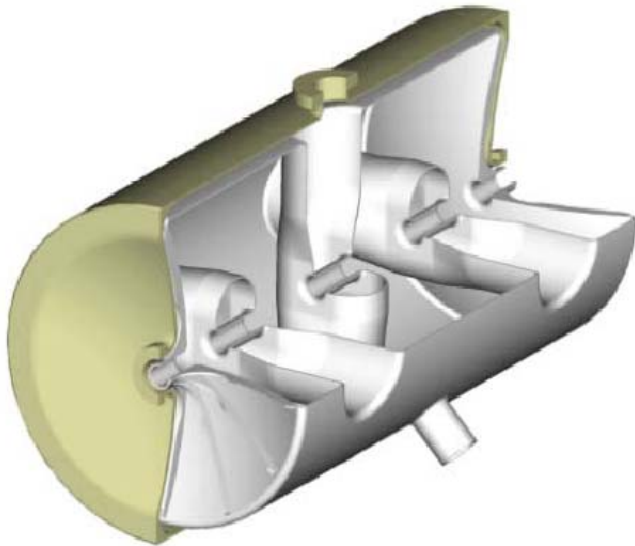
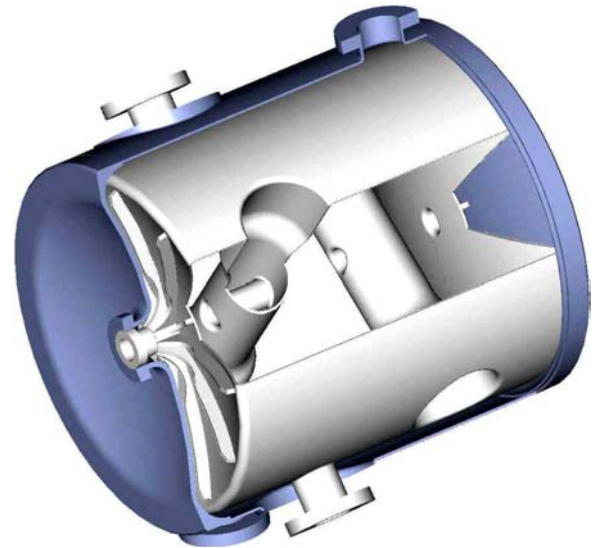
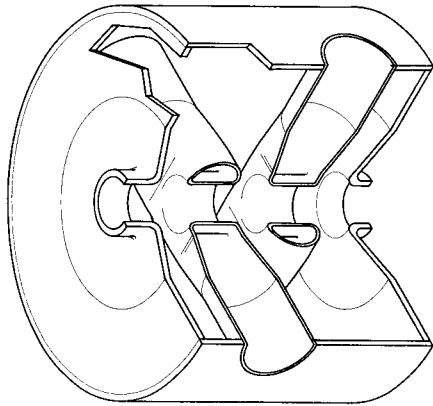


# $\lambda/2$ Resonant Lines – Single-Spoke



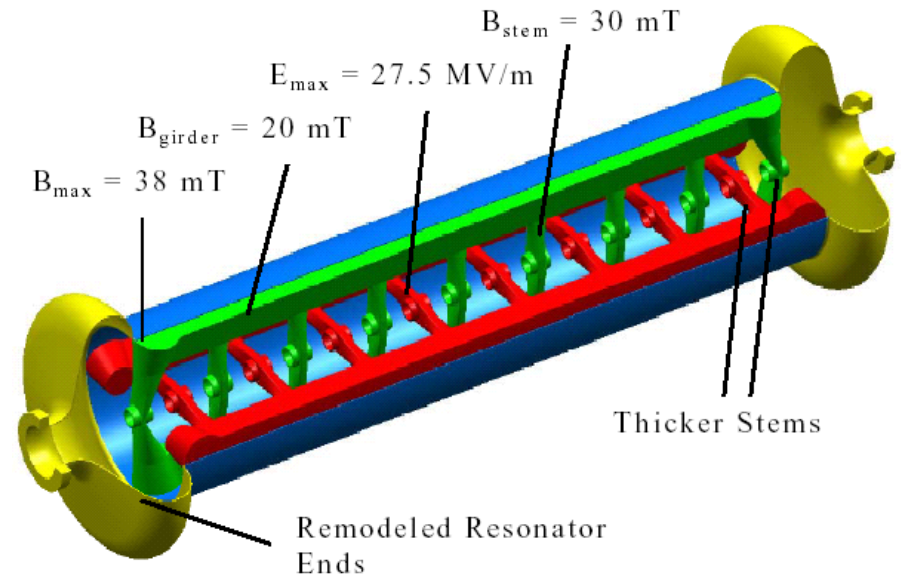


# $\lambda/2$ Resonant Lines – Double and Triple-Spoke

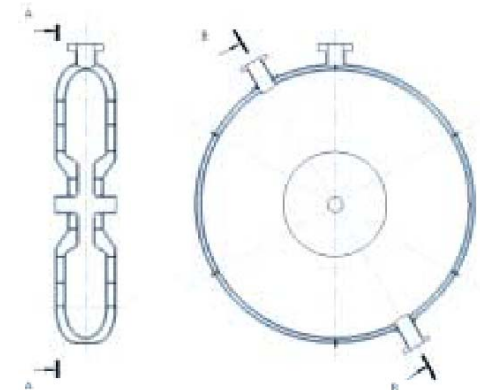




# $\lambda/2$ Resonant Lines – Multi-Spoke



# TM Modes



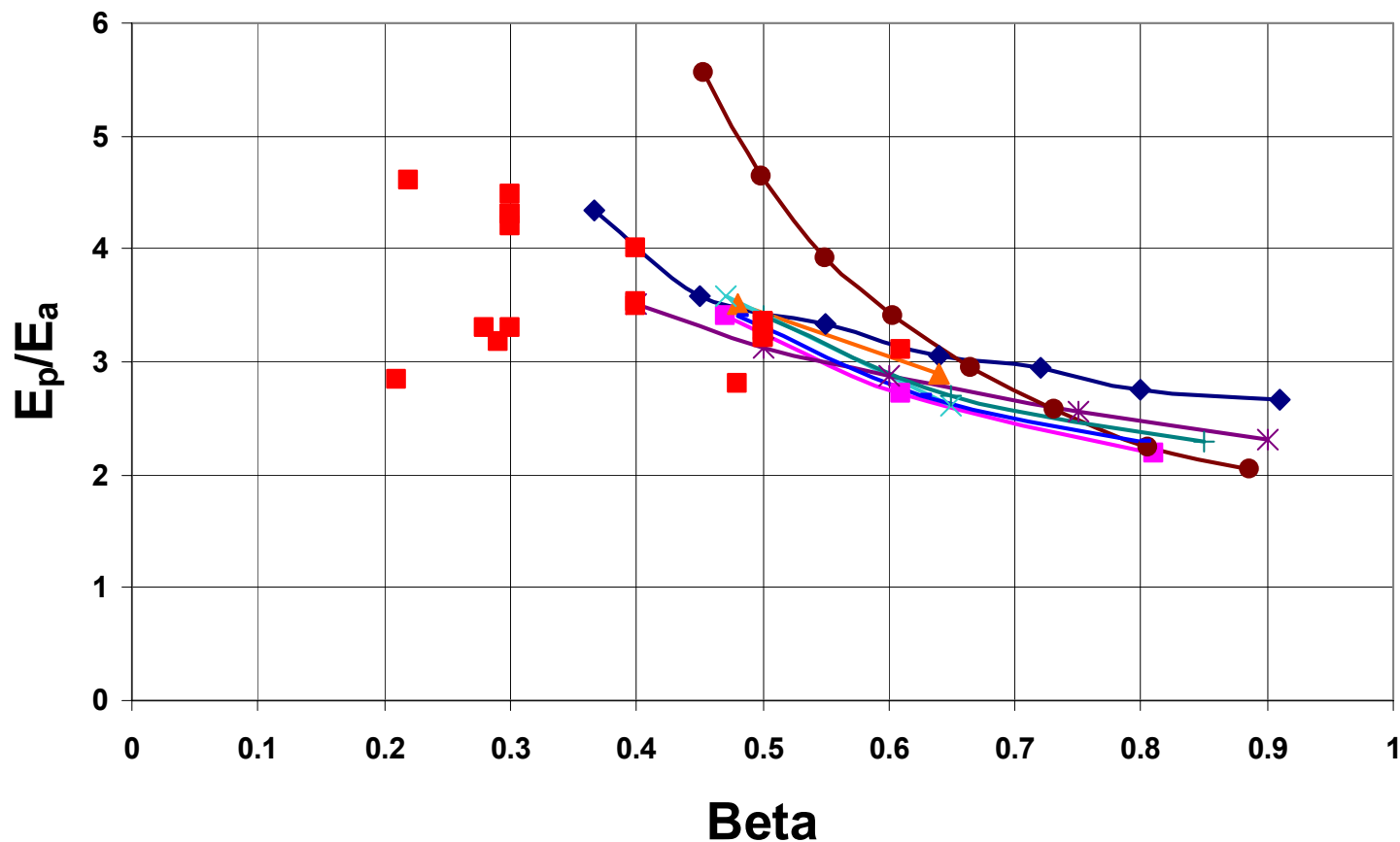
# Surface Electric Field

- $TM_{010}$  elliptical structures
  - $E_p/E_a \sim 2$  for  $\beta = 1$
  - Increases slowly as  $\beta$  decreases
- $\lambda/2$  structures:
  - Sensitive to geometrical design
  - Electrostatic model of an “shaped geometry” gives  $E_p/E_a \sim 3.3$ , independent of  $\beta$



# Surface Electric Field

- Lines: Elliptical
- Squares: Spoke



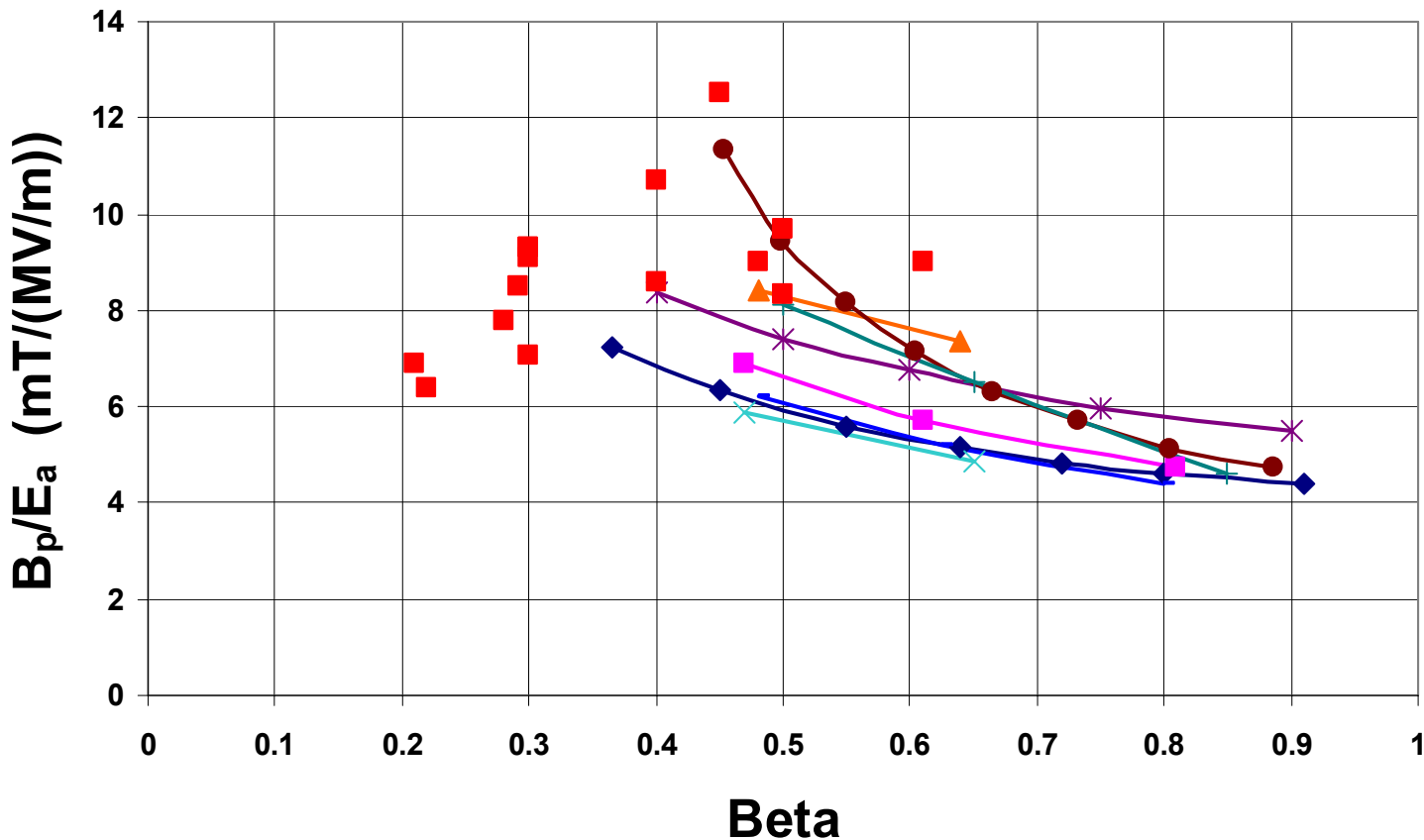
# Surface Magnetic Field

- $TM_{010}$  elliptical cavities:
  - $B/E_a \sim 4 \text{ mT}/(\text{MV}/\text{m})$  for  $\beta=1$
  - Increases slowly as  $\beta$  decreases
- $\lambda/2$  structures:
  - Sensitive to geometrical design
  - Transmission line model gives  $B/E_a \sim 8 \text{ mT}/(\text{MV}/\text{m})$ , independent of  $\beta$



# Surface Magnetic Field

- Lines: Elliptical
- Squares: Spoke





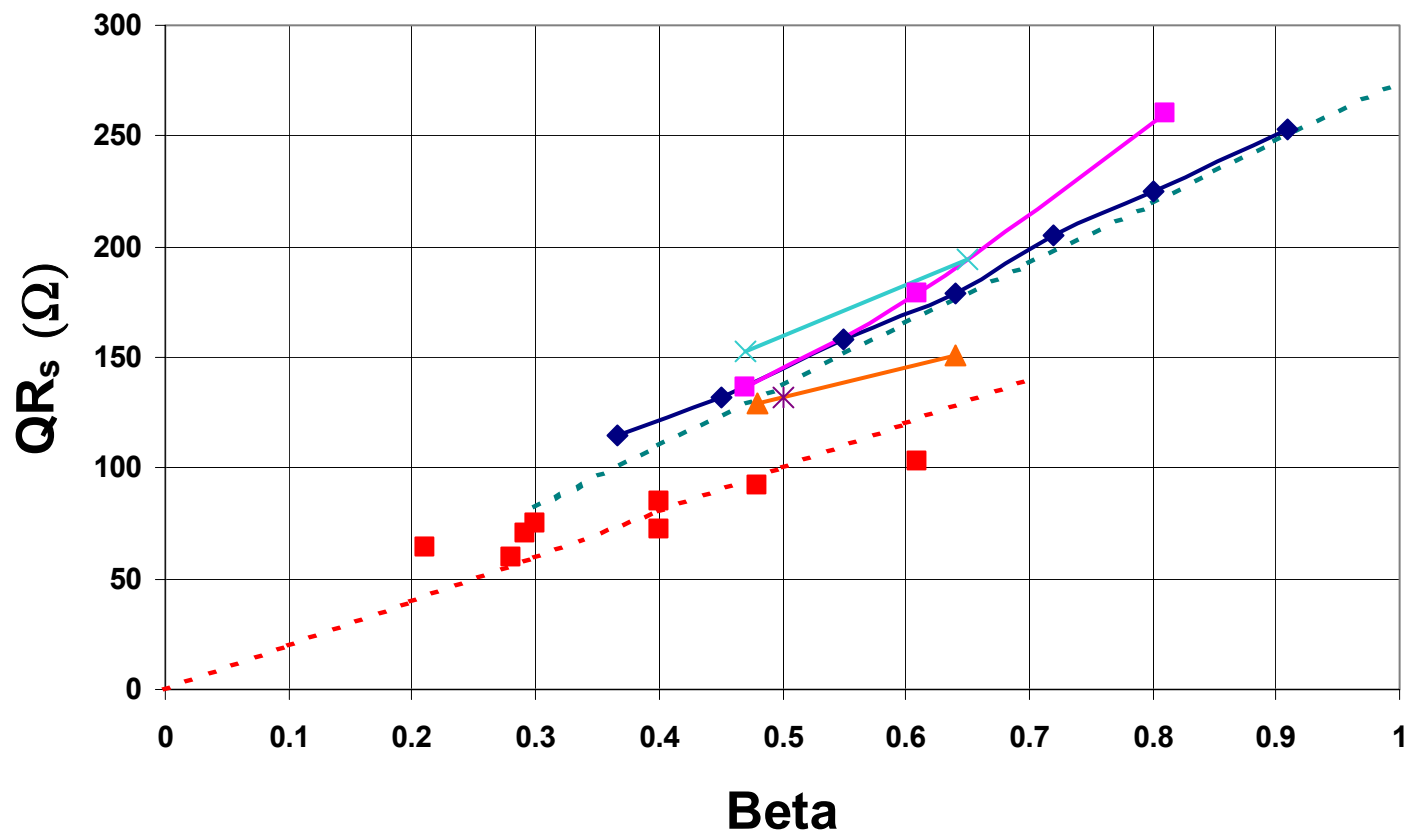
# Geometrical Factor ( $QR_s$ )

- $TM_{010}$  elliptical cavities:
  - Simple scaling:  $QR_s \sim 275 \beta \ (\Omega)$
- $\lambda/2$  structures:
  - Transmission line model:  $QR_s \sim 200 \beta \ (\Omega)$



# Geometrical Factor ( $QR_s$ )

- Lines: Elliptical
- Squares: Spoke



# $R_{sh}/Q$ per cell or loading element

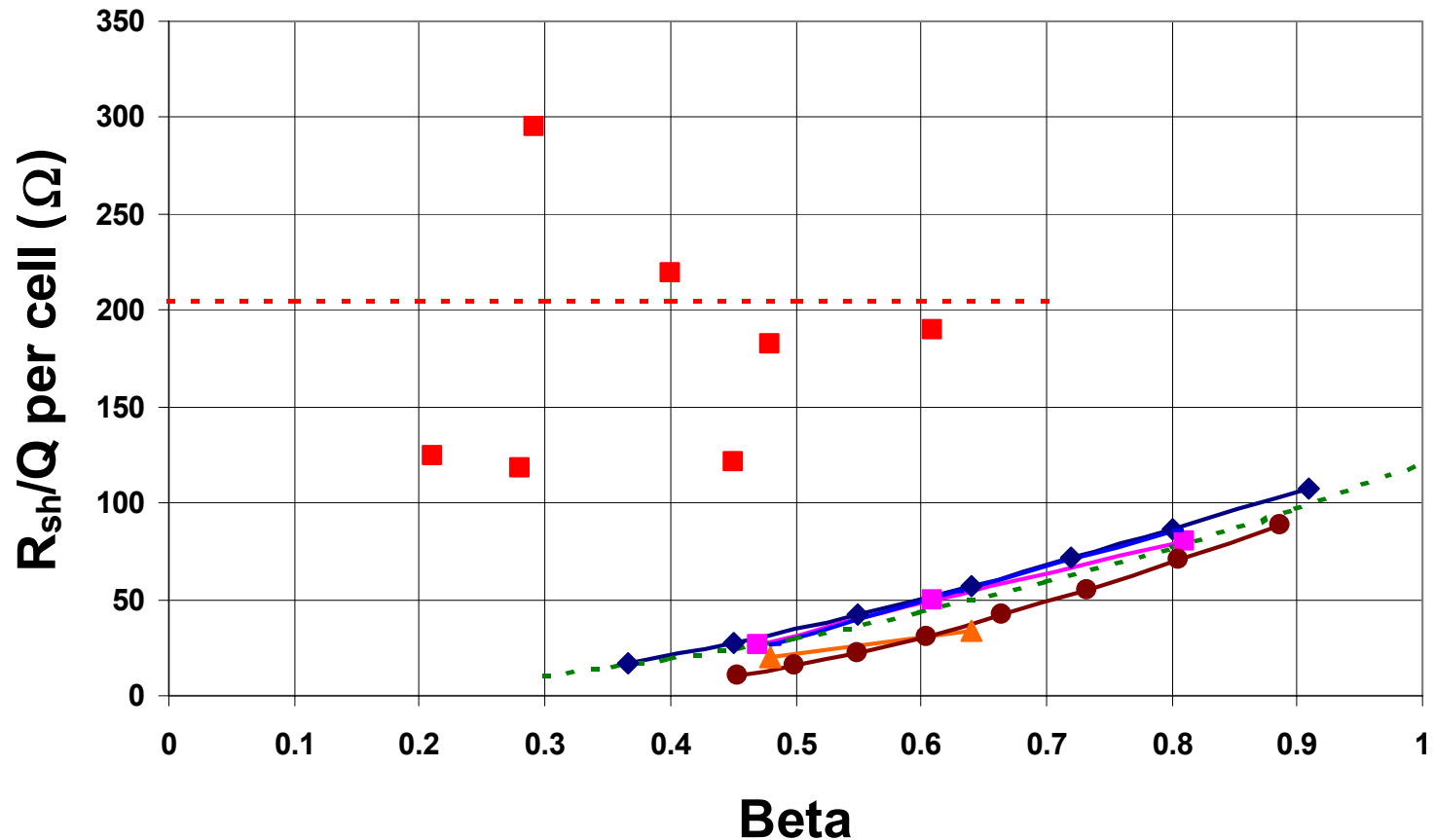
- $R_{sh} = V^2/P$
- TM<sub>010</sub> elliptical cavities:
  - Simple-minded argument, ignoring effect of beam line aperture, gives :  $R_{sh}/Q \propto \beta$
  - When cavity length becomes comparable to beam line aperture :  
 $R_{sh}/Q \propto \beta^2$
  - $R_{sh}/Q \approx 120 \beta^2 \text{ } (\Omega)$
- $\lambda/2$  structures:
  - Transmission line model gives:  $R_{sh}/Q \approx 205 \text{ } \Omega$
  - Independent of  $\beta$



# $R_{sh}/Q$ per cell or loading element

Lines: Elliptical

Squares: Spoke



# Shunt Impedance $R_{sh}$ ( $R_{sh}/Q$ $QR_s$ per cell or loading element)

- $TM_{010}$  elliptical cavities:
  - $R_{sh} R_s \sim 33000 \beta^3 (\Omega^2)$
- $\lambda/2$  structures:
  - $R_{sh} R_s \sim 40000 \beta (\Omega^2)$

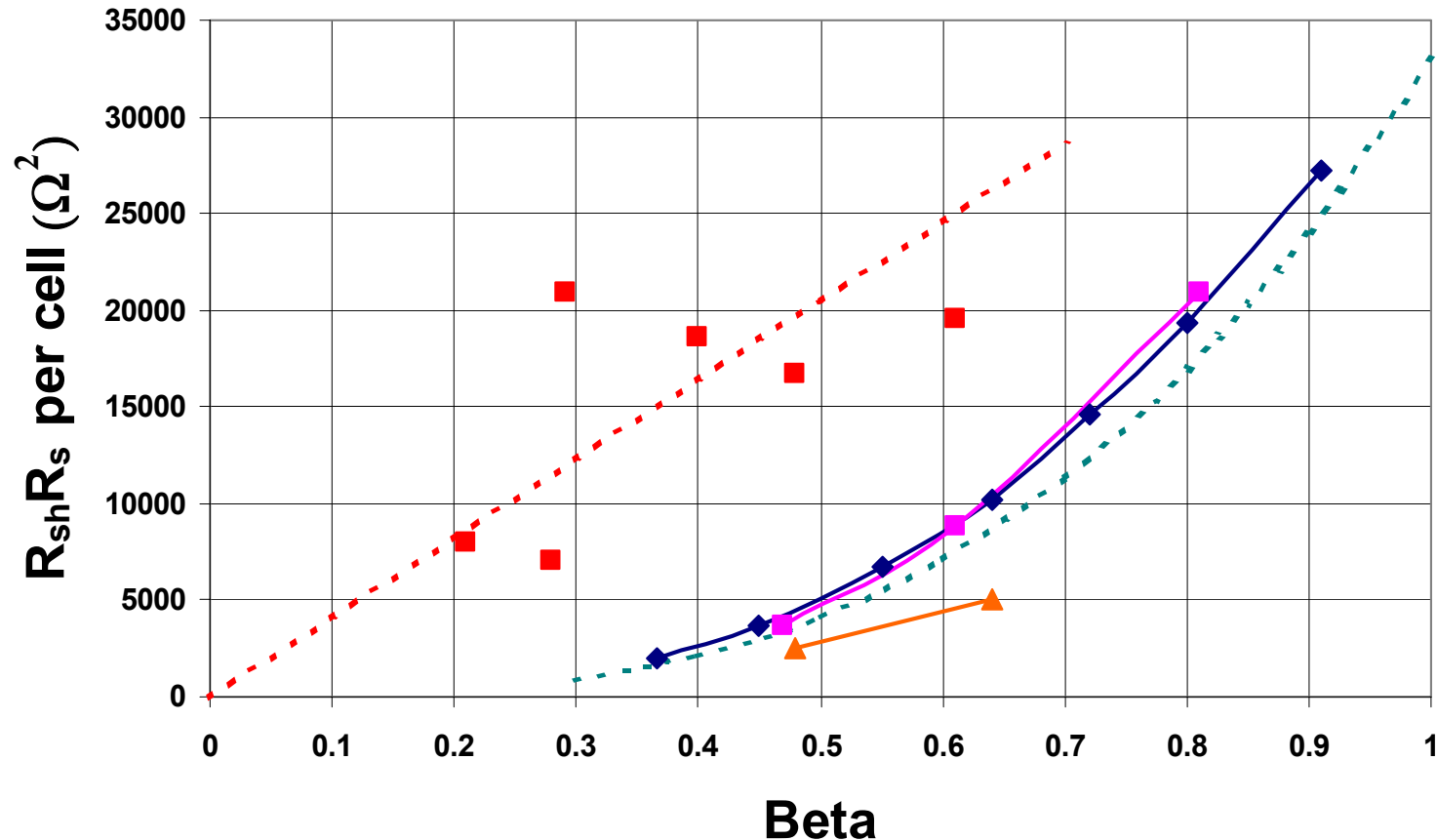


# Shunt Impedance $R_{sh}$

( $R_{sh}/Q$   $QR_s$  per cell or loading element)

• Lines: Elliptical

Squares: Spoke





# Energy Content per Cell or Loading Element

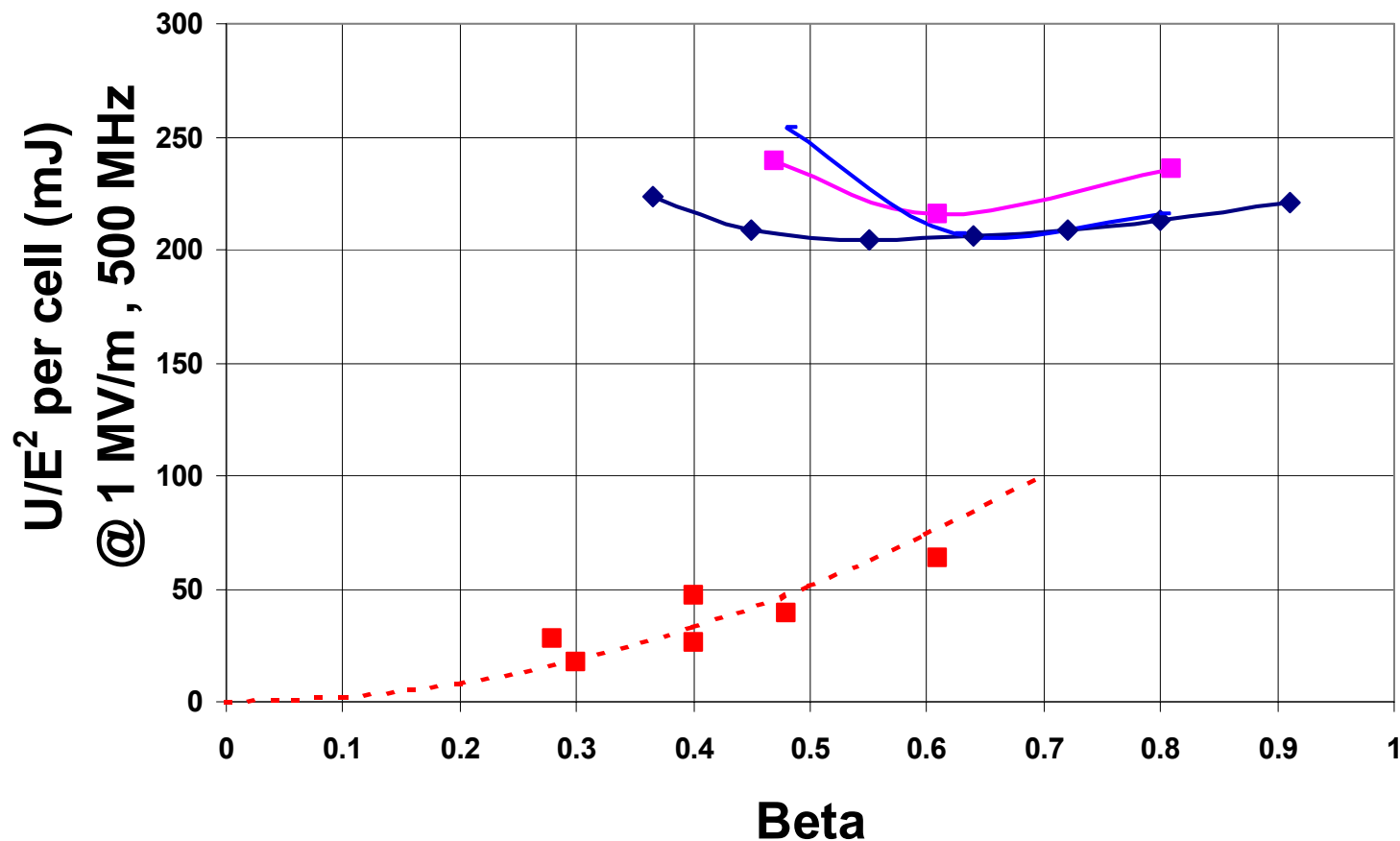
Proportional to  $E^2\lambda^3$

At 1 MV/m, normalized to 500 MHz:

- $TM_{010}$  elliptical cavities:
  - Simple-minded model gives  $U/E^2 \propto \beta$
  - In practice:  $U/E^2 \sim 200\text{-}250 \text{ mJ}$
  - Independent of  $\beta$  (seems to increase when  $\beta < 0.5 - 0.6$ )
- $\lambda/2$  structures:
  - Sensitive to geometrical design
  - Transmission line model gives  $U/E^2 \sim 200 \beta^2 \text{ (mJ)}$



# Energy Content per Cell or Loading Element

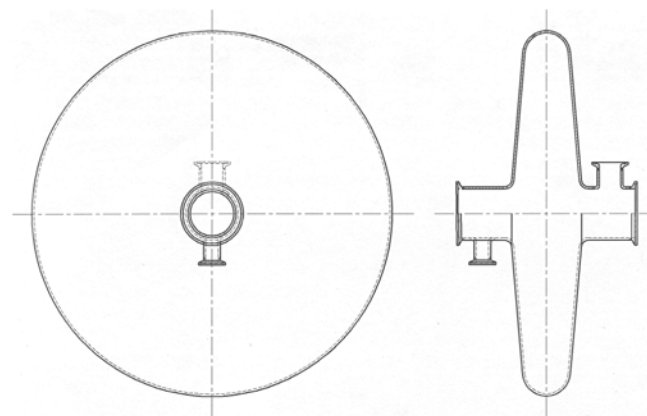


# Size & Cell-to-Cell Coupling

- $TM_{010}$  Structures

$$\varnothing \sim 0.88 - 0.92 \lambda$$

Coupling  $\sim 2\%$

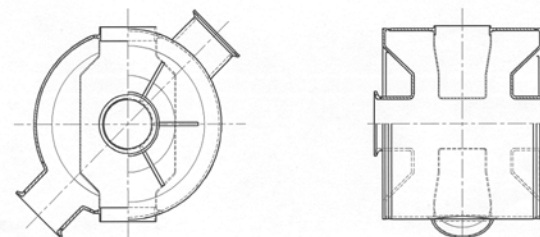


10 20 30  
Scale in cm

- $\lambda/2$  Structures

$$\varnothing \sim 0.46 - 0.51 \lambda$$

Coupling  $\sim 20 - 30\%$



Example : 350 MHz,  $\beta = 0.45$



# Multipacting

- $TM_{010}$  elliptical structures
  - Can reasonably be modeled and predicted/avoided
  - Modeling tools exist
- $\lambda/2$  Structures
  - Much more difficult to model
  - Reliable modeling tools do not exist
  - Multipacting “always” occurs
  - “Never” a show stopper



# TM Structures – Positive Features

- Geometrically simple
- Familiar
- Large knowledge base
- Good modeling tools
- Low surface fields at high  $\beta$
- Small number of degrees of freedom



# $\lambda/2$ Structures – Positive Features

- Compact, small size
- High shunt impedance
- Robust, stable field profile (high cell-to-cell coupling)
- Mechanically stable, rigid (low Lorentz coefficient, microphonics)
- Small energy content
- Low surface fields at low  $\beta$
- Large number of degrees of freedom





# What Next?

- How high in  $\beta$  can spoke cavities go?
- What are their high-order modes properties?
  - Spectrum
  - Impedances
  - Beam stability issues
- Is there a place for spoke cavities in high- $\beta$  high-current applications?
  - FELs, ERLs
  - Higher order modes extraction

